3rd International Conference on Industrial Engineering and Industrial Management
XIII Congreso de Ingeniería de Organización
Barcelona-Terrassa, September 2nd-4th 2009

Using Tabu Search for the Response Time Variability Problem

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Keywords: response time variability, tabu search, scheduling, regular sequences

1. Introduction

The concept of fair sequence has emerged independently from scheduling problems of diverse environments. The common aim of these scheduling problems, as defined in Kubiak (2004), is to build a fair sequence using n symbols, where symbol i (i = 1,...,n) must occur d_i times in the sequence. The fair sequence is the one which allocates a fair share of positions to each symbol i in any subsequence. This fair or ideal share of positions allocated to symbol i in a subsequence of length k is proportional to the relative importance (d_i) of symbol i with respect to the total copies of competing symbols (equal to $\sum_{i=1..n} d_i$). There is no a universal definition of fairness because several reasonable metrics can be defined according to the specific problem considered.

Among the different definitions of fairness, several fair sequencing problems have emerged, among them the Response Time Variability Problem (RTVP). This problem has been first time reported in Waldspurger and Weihl (1994) and originally formalised in Corominas et al. (2007). In the RTVP, the fair sequence is the one which minimises the sum of the variability in the distances between any two consecutive copies of the same symbol. In other words, the distance between any two consecutive copies of the same symbol should be as regular as possible (ideally constant) .

The RTVP arises whenever products, clients or jobs need to be sequenced so as to minimize variability in the time between the instants at which they receive the necessary resources (Corominas et al., 2007). This problem has a broad range of real-world applications. These include, for instance, the sequencing on mixed-model assembly lines under JIT (Kubiak, 1993; Miltenburg, 1989), the resource allocation in computer multi-threaded systems such as operating systems, network servers and media-based applications (Dong et al., 1998; Waldspurger and Weihl, 1995), the periodic machine maintenance problem when the times between consecutive services of the same machine are equal (Anily et al., 1998; Wei and Liu, 1983), the collection of waste (Herrmann, 2007) and the schedule of commercial videotapes for television (Bollapragada et al., 2004; Brusco, 2008).

The RTVP is NP-hard (Corominas et al., 2007). Since this problem is a difficult combinatorial optimisation problem, several heuristic and metaheuristic algorithms has been proposed in the literature to solve it. Waldspurger and Weihl (1995) used the Jefferson method of apportionment (Balinski and Young, 1982), a greedy heuristic algorithm which they renamed as the stride scheduling technique. Herrmann (2007) solved the RTVP by

[*] Supported by the Spanish Ministry of Education and Science under project DPI2007-61905; co-funded by the ERDF.

applying a heuristic algorithm based on the stride scheduling technique. Corominas et al. (2007) proposed also the Jefferson method together with other four greedy heuristic algorithms. García et al. (2006) proposed six metaheuristic algorithms: a multi-start, a greedy randomized adaptive search procedure (GRASP) and four variants of a discrete particle swarm optimization (PSO) algorithm. An enhanced multi-start algorithm and an enhanced GRASP algorithm were proposed in Corominas et al. (2008), and other ten discrete PSO algorithms were proposed in García-Villoria and Pastor (2009a). A cross-entropy method approach was used in García-Villoria et al. (2007) and a psychoclonal algorithm was used to solve the RTVP in García-Villoria and Pastor (2008). Finally, an algorithm based on Electromagnetism-like Mechanism (EM) was proposed in García-Villoria and Pastor (2009b). The best results recorded to date have been obtained with the psychoclonal algorithm (García-Villoria and Pastor, 2008) and the enhanced multi-start algorithm (Corominas et al., 2008).

To date, no tabu search (TS) approach has been proposed to solve the RTVP. In this study we propose a TS algorithm for the RTVP which improves the best results reported in the literature.

The remainder of the paper is organized as follows: Section 2 presents a formal definition of the RTVP and describes briefly the two best algorithms up to now for solving the problem. Section 3 proposes a TS algorithm to solve the RTVP. Section 4 presents the results of a computational experiment. Finally, some conclusions and suggestions for future research are given in Section 5.

2. The Response Time Variability Problem

The RTVP is formulated as follows. Let n be the number of symbols, d_i the number of copies to be sequenced of symbol i (i = 1,…,n) and D the total number of copies ($\sum_{i=1..n} d_i$). Let s be a solution of an instance in the RTVP that consists of a circular sequence of copies ($s = s_1 s_2 \dots s_D$), where s_j is the copy sequenced in position j of sequence s. For each symbol i in which $d_i \geq 2$, let t_k^i be the distance between the positions in which the copies k + 1 and k of symbol i are found. We consider the distance between two consecutive positions to be equal to 1. Since the sequence is circular, position 1 comes immediately after position D; therefore, $t_{d_i}^i$ is the distance between the first copy of symbol i in a cycle and the last copy of the same symbol in the preceding cycle. Let $\overline{t_i}$ be the desired average distance between two consecutive copies of symbol i ($\overline{t_i} = D/d_i$). The objective is to minimise the metric called response time variability (RTV), which is defined by the sum of the square errors with respect to the $\overline{t_i}$ distances. Since the symbols i such that $d_i = 1$ do not intervene in the computation of RTV, we assume that for each of these symbols t_1^i is equal to $\overline{t_i}$. Thus, RTV is given by the following expression:

$$RTV = \sum_{i=1}^{n} \sum_{k=1}^{d_i} \left(t_k^i - \overline{t_i} \right)^2 . \tag{1}$$

For example, let $n = 3$ with symbols A, B and C. Also consider $d_A = 2$, $d_B = 2$ and $d_C = 4$; thus, $D = 8$, $\overline{t_A} = 4$, $\overline{t_B} = 4$ and $\overline{t_C} = 2$. Any sequence such that contains symbol i ($\forall i$) exactly d_i times is a feasible solution. For example, the sequence (C, A, C, B, C, B, A, C) is a feasible solution, and has an $RTV = \left((5-4)^2 + (3-4)^2\right) + \left((2-4)^2 + (6-4)^2\right) + \left((2-2)^2 + (2-2)^2 + (3-2)^2 + (1-2)^2\right)$ = 12.

As it has been introduced in Section 1, the psychoclonal algorithm proposed in García-Villoria and Pastor (2008) and the multi-start algorithm proposed in Corominas et al. (2008) are the best procedures to date to solve the RTVP.

Psychoclonal is an evolutionary metaheuristic first time proposed in Tiwari et al. (2005). According to the authors, this metaheuristic inherits its characteristics from the need hierarchy theory of Maslow (1954) and the clonal selection principle (Gaspar and Collard, 2000). The basic scheme of the psychoclonal metaheuristic is the following: 1) An initial population of solutions is generated and a function to evaluate the fitness of a solution is given; 2) The best solutions are selected and cloned in a number proportional to their fitness; 3) The generated clones are hypermutated (hypermutation is an operator that modifies the solution with a rate inversely proportional to the fitness of the solution); 4) A new population is formed by the best clones and by new solutions generated at random; 5) Steps 2-4 are repeated until a stop condition is reached. This metaheuristic was adapted to solve the RTVP (for a more detailed explanation, see García-Villoria and Pastor, 2008).

The general scheme of the multi-start metaheuristic consists of two phases. In the first phase an initial solution is generated. Then, the second phase improves the obtained initial solution. These two phases are iteratively applied until a stop condition is reached. Thus, the multi-start algorithm proposed in Corominas et al. (2008) to solve the RTVP consists of, at each iteration, generating an initial solution by a random mechanism (first phase) and then applying it a local search (second phase); the stop condition consists in reaching a given computing time (for a more detailed explanation, see Corominas et al., 2008)

3. A Tabu Search algorithm to solve the RTVP

Local search methods have the great disadvantage that the local optimum found is often a fairly mediocre solution (Gendreau, 2003). To overcome this limitation, the Tabu Search metaheuristic (TS) has been proposed by Glover (1986). TS is based on applying a local search in which non-improving movements are allowed. To avoid cycling back to visited solutions, the most recent history of the search is recorded in a tabu list of tabu (forbidden) solutions. The complete tabu solutions could be recorded in the tabu list, but this may require a lot of memory, make it expensive to check whether a solution is tabu or not and, above all, does not diversify sufficiently the search. Thus, it is common to record only the last moves (transformations) performed on the current solution and forbidding reverse transformations (Gendreau, 2003). The tabu lists are usually implemented as a list of fixed length with a FIFO (First In, First Out) policy. A tabu solution can be overridden if a suitable aspiration criterion is met. The general scheme of TS is show in Figure 1.

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1. Define the neighbourhood structure N
2. Let S an initial solution and S* := S
3. While stopping condition is not reached do:
4.       Let S’ the best solution from N(S) which is non-tabu or allowed by aspiration
5.       If S’ is better than S*, then S* := S’
6.       Add the current move in the tabu list (removing its last move if the list is full)
7.       S := S’
8. End While
9. Return S*
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Figure 1. General scheme of TS

We propose an algorithm based on the general scheme of TS to solve the RTVP. The elements of the proposed TS algorithm are defined as follows:

– Initial solution. A solution is represented by the sequence of the copies of the symbols to be sequenced. The initial solution is obtained from the best solution returned by the five heuristics proposed in Corominas et al. (2007).

– Neighbourhood. The neighbourhood of a solution is obtained by swapping each pair of consecutive or non-consecutive positions of the sequence that represents the solution.

– Tabu moves. A forbidden move of the tabu list consists of two pairs of position/symbol. For instance, the move [(3, A), (5, B)] means that all solutions with the symbol A sequenced in position 3 and the symbol B sequenced in position 5 are considered tabu.

– Aspiration criterion. The aspiration criterion is that the move produces a solution better than the best solution found in the past.

– Stopping condition. The TS algorithm stops once it has run for a preset time.

The TS algorithm has only one parameter whose value has to be set and it is the size of the tabu list. Although the parameter values are extremely important because the results of the metaheuristic for each problem are very sensitive to them, the selection of parameter values is commonly justified in one of the following ways (Eiben et al., 1999; Adenso-Díaz and Laguna, 2006): 1) "by hand" on the basis of a small number of experiments that are not specifically referenced; 2) by using the general values recommended for a wide range of problems; 3) by using the values reported to be effective in other similar problems; or 4) by choosing values without any explanation.

Adenso-Díaz and Laguna (2006) proposed a new technique called CALIBRA specifically designed for fine-tuning the parameters of heuristic and metaheuristic algorithms. CALIBRA was used in García-Villoria and Pastor (2008) and in Corominas et al. (2008) to set the parameter values of the psychoclonal and the multi-start algorithms, respectively. We used also CALIBRA to set the size of the tabu list of our TS algorithm. CALIBRA was applied to a training set. The training set has 60 instances which were generated as explained in the introduction of Section 4. The optimal value of the size tabu list returned by CALIBRA was 75.

4. Computational experiment

The psychoclonal and the multi-start algorithms proposed in García-Villoria and Pastor (2008) and in Corominas et al. (2008), respectively, are the most efficient algorithms in the literature to solve the RTVP. Therefore, we compare the performance of our proposed TS algorithm with these two algorithms. In what follows of this section, we refer to our TS algorithm as TS, the psychoclonal algorithm as Psycho and the multi-start algorithm as MS.

All algorithms are coded in Java and executed on a 3.4 GHz Pentium IV with 1.5 GB of RAM. The same 60 training instances and 740 test instances used in García-Villoria and Pastor (2008) and in Corominas et al. (2008) are also used in this paper. These instances were grouped into four classes (from CAT1 to CAT4 with 15 training instances and 185 test instances in each class) according to their size. The instances were generated using the random values of D (total number of copies) and n (number of symbols) shown in Table 1. For all instances and for each symbol $i = 1,\ldots,n$, a random value of d_i (number of copies to be sequenced of model i) is between 1 and $\left|(D-n+1)/2.5\right|$ such that $\sum_{i=1..n} d_i = D$.

Table 1. Uniform distribution for generating the D and n values

	CAT1	CAT2	CAT3	CAT4
D	U(25, 50)	U(50, 100)	U(100, 200)	U(200, 500)
n	U(3, 15)	U(3, 30)	U(3, 65)	U(3, 150)

The algorithms were run for 50 and 1,000 seconds for each instance. Table 2 and Table 3 shows the overall average RTV values for the 740 test instances and for each class of instances (CAT1 to CAT4) obtained with the three algorithms, respectively.

Table 2. Average RTV values for a computing time of 50 seconds

	Global	CAT1	CAT2	CAT3	CAT4
TS	202.42	10.30	22.40	109.38	667.59
Psycho	235.68	14.92	44.25	137.07	746.50
MS	2,106.01	11.56	38.02	154.82	8,219.65

Table 3. Average RTV values for a computing time of 1,000 seconds

	Global	CAT1	CAT2	CAT3	CAT4
TS	113.31	10.24	21.46	106.21	315.33
Psycho	161.60	14.90	39.90	122.38	469.23
MS	169.25	10.51	31.21	123.27	512.02

Tables 2 and 3 shows that the multi-start algorithm converges much slower than the other two algorithms when big instances (CAT4 instances) are solved (Figure 2 shows how the algorithms converge during the computing time). Therefore, we analyse the results obtained by the algorithms after 1,000 seconds of computing time.

The global average RTV values of TS with 1,000 computing time seconds for all test instances are 29.88% and 33.05% better than the results obtained using Psycho or MS, respectively. If we consider the results by class, we can see that MS performs better than Psycho for the two smallest instances (CAT1 and CAT2), both performs very similar for the medium instances (CAT3) and Psycho performs better than MS for the biggest instances (CAT4). On the other hand, TS performs better than the other two algorithms for all type of instance: TS is 2.57% and 31.24% better than MS for CAT1 and CAT2 instances, respectively, and 13.21% and 32.80% better than Psycho for CAT3 and CAT4 instances, respectively.

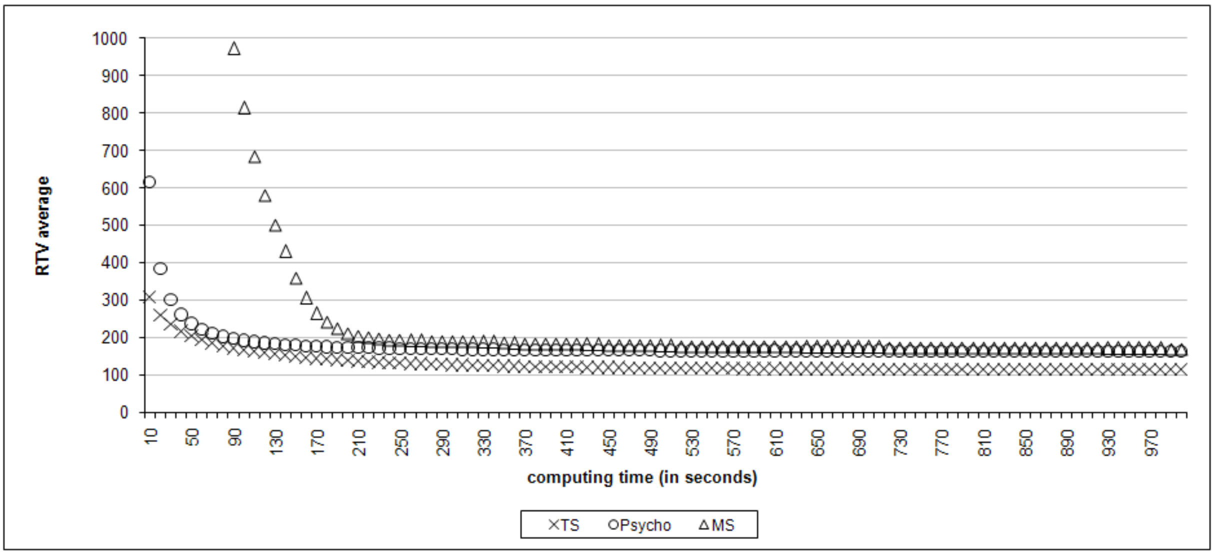

Figure 2. Average RTV values over the computing time

Table 4 shows the number of times that each algorithm reaches the best RTV value for each instance obtained using the three algorithms. The results are shown for the 740 instances overall and for each class of instance.

Table 4. Number of times that the best solution is reached

	Global	CAT1	CAT2	CAT3	CAT4
TS	587	185	185	113	104
Psycho	104	52	7	37	8
MS	305	164	18	48	75

As could be expected from the results in Table 3, Table 4 shows that TS reaches the best solution the greatest number of times (in 79.32% of the instances overall). Moreover, if the results are observed by class, it can be seen that TS always obtains the best solution for CAT1 and CAT2 instances. On the other hand, MS obtains more times the best solution than Psycho, even for the CAT4 instances. This is surprising since MS obtains a worst RTV values average for CAT4 instances than Psycho.

To complete the analysis of the results, we examined the dispersion of the results. A measure of the dispersion (let it be called σ) of the RTV values obtained by each algorithm alg = {TS, Psycho, MS} is defined for a given instance, ins, according to the following expression:

$$\sigma(alg, ins) = \left(\frac{RTV_{ins}^{(alg)} - RTV_{ins}^{(best)}}{RTV_{ins}^{(best)}} \right)^2 \quad (2)$$

where $RTV_{ins}^{(alg)}$ is the RTV value of the solution obtained with the algorithm alg for the instance ins, and $RTV_{ins}^{(best)}$ is the best RTV value of the solutions obtained with the three algorithms for the instance ins. Table 5 shows the average σ dispersion for the total number of instances and for each class.

Table 5. Average σ dispersion with respect to the best solution found

	Global	CAT1	CAT2	CAT3	CAT4
TS	0.42	0.00	0.00	0.21	1.18
Psycho	1.90	1.08	1.68	0.19	4.63
MS	0.48	0.02	0.43	0.19	1.30

Table 5 shows that TS has the lowest average σ dispersions for the total number of cases and for each instance class (except for CAT3, in which it is also low but slightly worse than the dispersions of the other two algorithms). That is, when TS does not obtain the best RTV value for a given instance, it obtains a value that is very close to it. MS has also a low dispersion for each instance class. On the other hand, Psycho has a quite worst dispersion for CAT4 instances than MS, although the RTV average obtained by Psycho for the CAT4 instances is better than the RTV average obtained by MS. This means that although Psycho obtains a better performance, on average, for the CAT4 instances, the MS is a more robust algorithm than Psycho. Anyway, the TS algorithm that we propose is the one that obtains, on average, the better solutions and the one that has the most stable behaviour.

5. Conclusions and future lines of research

The RTVP is a scheduling problem which has a broad range of real-world applications. Since the RTVP is NP-hard, several heuristic and metaheuristics have been proposed to solve it. Among them, the two algorithms with which the best results have been achieved are the psychoclonal algorithm proposed in García-Villoria and Pastor (2008) and the multi-start algorithm proposed in Corominas et al. (2008). We propose a straight application of the TS metaheuristics to solve the RTVP. The computational experiment shows that the proposed TS algorithm improves by far the best results published in the literature. Moreover, the TS algorithm is very stable; that is, when it does not obtain the best RTV value for a given instance, it obtains a value that is very close to it.

The definition of the neighbourhood structure is a very critical decision in the design of any TS algorithm (Gendreau et al., 2003). In this study we propose to generate the neighbourhood of a solution by swapping each pair of consecutive or non-consecutive positions of the solution sequence. Because of the good performance of the TS algorithm, a promising line of research is testing other neighbourhood structures in the proposed algorithm to solve the RTVP. Other candidate ways of generating the neighbourhood of a solution are, for example: 1) by swapping each pair of only consecutive positions of the sequence, and 2) by inserting each position in the sequence.

References

Adenso-Díaz, B., Laguna, M. (2006). Fine-tuning of algorithms using fractional experimental designs and local search. Operations Research, Vol. 54, pp. 99-114.

Anily, S., Glass, C.A., Hassin, R. (1998). The scheduling of maintenance service. Discrete Applied Mathematics, Vol. 82, pp. 27-42.

Balinski, M.L. and Young, H.P. (1982). Fair Representation: meeting the ideal of one man, one vote. Yale University Press, New Haven CT.

Bollapragada, S., Bussieck, M.R., Mallik, S. (2004). Scheduling Commercial Videotapes in Broadcast Television. Operations Research, Vol. 52, pp. 679-689.

Brusco, M.J. (2008). Scheduling advertising slots for television. Journal of the Operational Research Society, Vol. 59, pp. 1363-1372.

Corominas, A., Kubiak, W., Moreno, N. (2007). Response time variability. Journal of Scheduling, Vol. 10, pp. 97-110.

Corominas, A., García-Villoria, A., Pastor, R. (2008). Solving the Response Time Variability Problem by means of Multi-start and GRASP metaheuristics. Special Issue of Frontiers in Artificial Intelligence and Applications on Artificial Intelligence Research and Development, Vol. 184, pp. 128-137.

Dong, L., Melhem, R., Mosse, D. (1998). Time slot allocation for real-time messages with negotiable distance constrains requirements. Fourth IEEE Real-Time Technology and Applications Symposium (RTAS'98), Denver, CO. pp. 131-136.

Eiben, A.E., Hinterding, R., Michalewicz, Z. (1999). Parameter control in evolutionary algorithms. IEEE Transactions on evolutionary computation, Vol. 3, pp. 124-141.

García, A., Pastor, R., Corominas, A. (2006). Solving the Response Time Variability Problem by means of metaheuristics. Special Issue of Frontiers in Artificial Intelligence and Applications on Artificial Intelligence Research and Development, Vol. 146, pp.187-194.

García-Villoria, A., Pastor, R., Corominas, A. (2007). Solving the Response Time Variability Problem by means of the Cross-Entropy Method. Special Issue on Production Line Systems: Concepts, Methods and Applications of the International Journal of Manufacturing Technology and Management, to be published.

García-Villoria, A., Pastor, R. (2008). Solving the Response Time Variability Problem by means of a psychoclonal approach. Journal of Heuristics, doi:10.1007/s10732-008-9082-2.

García-Villoria, A., Pastor, R. (2009a). Introducing dynamic diversity into a discrete particle swarm optimization. Computers & Operations Research, Vol. 36, pp. 951-966.

García-Villoria, A., Pastor, R. (2009b). Solving the Response Time Variability Problem by means of the Electromagnetism-like Mechanism. International Journal of Production Research, doi: 10.1080/00207540902862545.

Gaspar, A., Collard, P. (2000). Two models of immunization for time dependent optimization. IEEE International Conference on Systems Manufacturing and Cybernetics, pp. 113-118.

Gendreau, M. (2003). An Introduction to Tabu Search. Chapter 2 in Handbook of Metaheuristics, Eds. Glover and Kochenberger, Kluwer Academic Publishers, pp. 37-54.

Glover, F., (1986). Future paths for Integer Programming and Links to Artificial Intelligence. Computers and Operations Research, Vol. 5, pp. 533-549.

Herrmann, J.W. (2007). Generating Cyclic Fair Sequences using Aggregation and Stride Scheduling. Technical Report TR 2007-12, University of Maryland, USA. Available at http://hdl.handle.net/1903/7082.

Kubiak, W. (1993). Minimizing variation of production rates in just-in-time systems: A survey. European Journal of Operational Research, Vol. 66, pp. 259-271.

Kubiak, W. (2004). Fair Sequences. Chapter 19 in Handbook of Scheduling: Algorithms, Models and Performance Analysis, Chapman and Hall.

Maslow, A.H. (1954). Motivation and personality. New York: Harper & Bros.

Miltenburg, J. (1989). Level schedules for mixed-model assembly lines in just-in-time production systems. Management Science, Vol. 35, pp. 192-207.

Tiwari, M.K., Prakash, A., Kumar, A., Mileham, A.R. (2005). Determination of an optimal sequence using the psychoclonal algorithm. ImechE, Part B: Journal of Engineering Manufacture, Vol. 219, pp. 137-149.

Waldspurger, C.A., Weihl, W.E. (1994). Lottery Scheduling: Flexible Proportional-Share Resource Management. First USENIX Symposium on Operating System Design and Implementation.

Waldspurger, C.A., Weihl, W.E. (1995). Stride Scheduling: Deterministic Proportional-Share Resource Management. Technical Report MIT/LCS/TM-528, Massachusetts Institute of Technology, MIT Laboratory for Computer Science. Available at https://eprints.kfupm.edu.sa/67117.

Wei, W.D., Liu, C.L. (1983). On a periodic maintenance problem. Operations Research Letters, Vol. 2, pp. 90-93.